

# EXCITATION OF INTERNAL GRAVITY WAVES BY PENETRATIVE CONVECTION

## CONSEQUENCES ON THE ROTATION OF EVOLVED LOW-MASS STARS

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**Abstract.** We investigate the ability of internal gravity waves that are generated by penetrative convection to redistribute angular momentum in the internal radiative zone of evolved low-mass stars. To do so, we use the semianalytical excitation model recently proposed by Pinçon *et al.* 2016. We briefly report the preliminary results of the study focusing on the subgiant and red giant branch stars.

### 1 Introduction

The space-borne missions CoRoT and *Kepler* provided us with thousands of seismic data and the detection of mixed modes made the measurement of the core rotation rate possible for subgiant and red giant stars (Mosser *et al.* 2012; Deheuvels *et al.* 2014). It turned out that they actually rotate more slowly than expected from stellar evolution codes including angular momentum transport by meridional circulation and shear-induced turbulence (Marques *et al.* 2013; Ceillier *et al.* 2013). Therefore, another process should operate in these stars, and internal gravity waves (hereafter, IGW) remain a serious candidate.

The efficiency of the transport by IGW depends on the energy of the waves, and so on the excitation mechanism. IGW can be generated by turbulent pressure in the convective zone (Kumar *et al.* 1999). While these waves were shown to be efficient in the solar case (Talon *et al.* 2002), Fuller *et al.* (2014) showed that they could not explain the low rotation rates observed in the core of more evolved stars. Nevertheless, penetrative convection at the base of the convective zone can also generate IGW. This mechanism has already been studied in geophysics (Townsend 1966) and in numerical simulations (*e.g.*, Dintrans 2005), but a theoretical estimate was still missing until the recent paper by Pinçon *et al.* (2016). In this latter work, these waves have notably been showed to be able on their own to efficiently modify the internal rotation in the solar case.

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In the following, we briefly describe the excitation model by penetrative convection and use it to estimate the energy of the convective plumes transferred into waves at the base of the convective zone. As a first application, we present preliminary results of the investigation about the angular momentum redistribution by these waves in low-mass evolved stars.

## 2 Wave flux induced by penetrative convection

Convective plumes are strong coherent downwards flows that originate from the uppermost layers of stars and that grow by turbulent entrainment of matter as they sink into the convective zone. Once they reach the bottom of the convective region, they can penetrate by inertia into the underlying stably stratified layers where they are slowed down by buoyancy braking (*e.g.* Zahn 1991) and transfer a part of their kinetic energy into waves. To model this, Pinçon *et al.* (2016) considered the pressure exerted by an ensemble of incoherent, uniformly distributed convective plumes as the driving mechanism in the wave equation. By assuming a high Péclet number and a very sharp thermal transition at the base of the convective zone, they derived a simplified expression for the mean radial wave energy flux per unit of frequency, for an angular degree  $l$  and an azimuthal number  $m$  at the top of the radiative zone,

$$\mathcal{F}_{E,w}(r_t, \omega, l, m) \sim \frac{1}{4\pi r_t^2} \frac{\mathcal{A} \mathcal{S}_p}{2} \frac{\rho_b V_b^3}{2} F_{R,l} \frac{e^{-\omega^2/4\nu_p^2}}{\nu_p} e^{-l(l+1)b^2/2r_t^2}, \quad (2.1)$$

where  $r_t$  is the radius at the top of the radiative zone,  $\mathcal{A}$  is the plumes filling factor in the excitation region,  $\mathcal{S}_p = \pi b^2$  is the horizontal area occupied by one single plume, with  $b$  the plume radius,  $\rho_b$  and  $V_b$  are respectively the density and the plume velocity at the base of the convective region,  $F_{R,l} = \sqrt{l(l+1)} V_b / r_t N_t$ , with  $N_t$  the Brunt-Väisälä frequency at the top of the radiative zone, and  $\nu_p = 1/\tau_p$ , with  $\tau_p$  the plume lifetime.

After having been generated, plume-induced IGW can propagate through the radiative zone of stars where they are damped by radiative diffusion (*e.g.* Press 1981). Here, they can deposit their angular momentum into the medium and so contribute to the transport of angular momentum in presence of differential rotation. Studying the transport of angular momentum by IGW requires to know the wave flux of angular momentum throughout the radiative zone of the star. This latter can be deduced from the sum of all the contributions given by Equation (2.1) and modulated by a radiative damping term. We refer the readers to previous works by Zahn (1997) and Pinçon *et al.* (2016)<sup>1</sup> for details.

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<sup>1</sup> Note that Equation (48) of 2016 contains a typo. It must be corrected by a factor  $r_d^2/r^2$  (here,  $r_d = r_t$ ) in order to conserve the flux through a spherical shell in the adiabatic case.

### 3 How efficient is the angular momentum transport by plume-induced IGW in evolved low-mass stars?

#### 3.1 Timescales comparison

Investigating the effect of IGW on the evolution of stellar rotation is a numerical challenge since it implies to cover dynamical timescales differing in several orders of magnitude. A simpler alternative is to compare the different timescales involved in the transport of angular momentum in order to discriminate the most efficient mechanism for different evolutionary stages. For example, the local timescale for IGW to modify a given rotation profile,  $\Omega(r)$ , can be estimated by

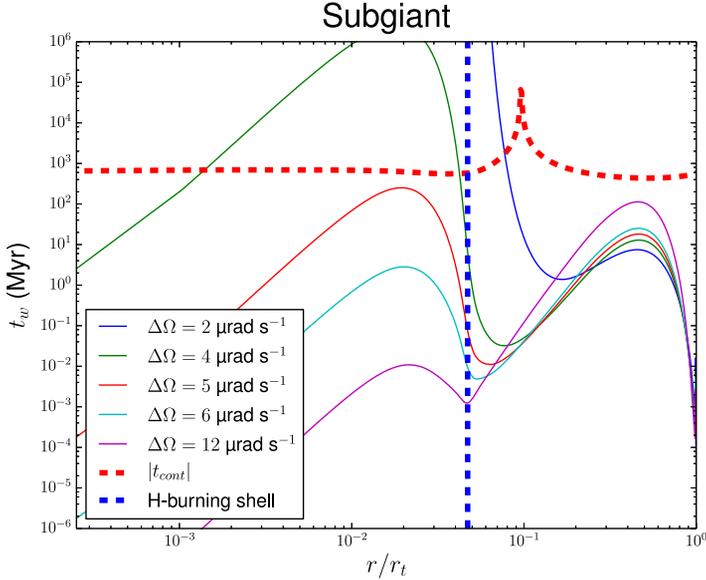
$$t_w(r) = \left| \frac{\rho(r)r^2\Omega(r)}{\dot{J}(r)} \right| \quad (3.1)$$

where  $r$  is the radius,  $\rho$  the density and  $\dot{J}$  is the radial divergence of the angular momentum wave flux (see Sect. 2 for its computation). Transport by IGW is then said to be more efficient than the acceleration due to the core contraction if  $t_w(r) < t_{cont}(r)$ , with  $t_{cont}$  the local contraction/dilatation timescale. Such a comparison provides us with a first indication about the effect of IGW on the evolution of the rotation.

As a first step, we used this simple diagnosis in two  $1 M_\odot$  standard models computed with the stellar evolution code CESTAM (Marques *et al.* 2013). Both are respectively located at the middle of the subgiant branch and at the beginning of the ascent of the RGB. To simplify, we assumed an arbitrary smooth shape for the differential rotation in the radiative region, in the form of  $\delta\Omega(r) = \Omega(r) - \Omega_t = \Delta\Omega \cos^2(\pi r/2r_t)$ , where  $\Omega_t$  is the value of the rotation rate at the top of the radiative zone and  $\Delta\Omega$  is the maximum amplitude. With all these ingredients, we could compute  $t_w$  and  $t_{cont}$  and compare them for both models. We varied  $\Delta\Omega$  between 0 and  $12 \mu\text{rad s}^{-1}$ , *i.e.* in a range in agreement with the observations (Mosser *et al.* 2012; Deheuvels *et al.* 2014).

#### 3.2 Star at the beginning of the ascent of the RGB

For the RGB star, we found that  $t_w \gg t_{cont}$  throughout the helium core located below the hydrogen-burning shell, whatever reasonable values for  $\Delta\Omega$ . Indeed, as pointed out by Fuller *et al.* (2014), the contraction of the core along the evolution results in an important increase of the Brunt-Väisälä frequency in the radiative zone of evolved stars. As a result, IGW are damped before reaching the hydrogen-burning shell, they cannot deposit angular momentum in the innermost layers and so modify the core rotation. We then conclude that IGW are unable on their own to counteract the acceleration due to the core contraction in low-mass RGB stars. Nevertheless, we do not exclude that the IGW that are strongly damped above the hydrogen-burning shell could interact with another transport process and indirectly enhance the angular momentum extraction from the core. This hypothesis will need to be investigated by a more exhaustive computation.



**Fig. 1.** Wave-driven timescale, Equation (3.1), as a function of the radius normalized by the radius at the top of the radiative zone, for a  $1 M_{\odot}$  subgiant model. The contraction timescale and the location of the hydrogen-burning shell are represented by the red and blue dashed lines, respectively. Different amplitudes for  $\Delta\Omega$  are considered.

### 3.3 Subgiant star

In the subgiant star, the wave-driven timescale is plotted in Figure 1 as a function of the radius. The situation is similar to the one in the RGB star for low values of  $\Delta\Omega$ : IGW are strongly damped above the hydrogen-burning shell and  $t_w \gg t_{cont}$  throughout the helium core. Nevertheless, as  $\Delta\Omega$  increases and becomes larger than about  $5 \mu\text{rad s}^{-1}$ , the wave-driven timescale becomes smaller than the contraction/dilatation timescale throughout the radiative zone. Indeed, as  $\Delta\Omega$  increases, the Doppler-shifted frequency of the retrograde waves increases, which in turn decreases their radiative damping. These waves are thus able to reach the core and slow down its rotation (since  $\dot{J} < 0$ ). Moreover, we note that the threshold value for  $\Delta\Omega$ , above which IGW can affect the core rotation, is very similar to the differential rotation observed in subgiants (Deheuvels *et al.* 2014). These results suggest that IGW generated by penetrative convection could be the missing ingredient to regulate the core rotation of subgiants in stellar modeling.

## 4 Conclusion

A semianalytical model of excitation by penetrative convection is now available. It enables us to get first indications on the role that plume-induced IGW can

play in the rotation history of stars. It turns out that neither the plume-induced IGW, nor turbulence-induced ones, may explain on their own the low rotation rates in RGB stars where another process should operate. Nevertheless, we find that IGW excited by penetrative convection may play a major role and regulate the core rotation in subgiants. These very preliminary results will have to be confirmed by a more thorough forthcoming study, considering different stellar masses, evolutionary stages and taking into account uncertainties on the parameters of the excitation model. In a further future, efforts will have also to be made to implement the transport by internal waves in a stellar evolution code with the interaction with other transport processes.

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