THE ROLE OF THE MAGNETOSONIC MACH NUMBER ON THE EVOLUTION OF KELVIN-HELMHOLTZ VORTICES

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Abstract. We review the main results of our previous works, in which we have investigated the development of the Kelvin-Helmholtz (KH) instability in the transitional regime from sub-magnetosonic to super-magnetosonic by varying the solar wind velocity, in conditions typical of those observed at the Earth’s magnetopause flanks. In super-magnetosonic regimes, we show that the vortices produced by the development of the KH instability act as an obstacle in the plasma flow and may generate quasi-perpendicular magnetosonic shock structures extending well outside the region of velocity shear.

1 Introduction

The connection between the magnetosheath and the Earth’s magnetosphere is mediated through the magnetopause boundaries. In the magnetosheath, the solar wind flows tangentially to the magnetopause and, through several mechanisms, can transfer energy and mass to the magnetosphere. In particular the transport of the energy and plasma is very efficient at the flanks of the magnetopause, where the KH instability and the complementary mechanism of magnetic reconnection can act. In these zones the interplanetary magnetic field (IMF) associated to the solar wind is often parallel to the geomagnetic field lines. When the magnetosphere and the magnetosheath field lines are in opposite direction magnetic reconnection dominates the transport properties of the plasma. On the contrary, when the IMF and the geomagnetic fields are in the same direction, the KH instability,
caused by the gradient of the plasma velocity generated between the solar wind flow and the static magnetospheric plasma, is the most relevant phenomenon, that can lead to plasma mixing (Fairfield et al. 2000). Considering the Earth’s frame, the solar wind enters the magnetosheath with a subsonic velocity. However, as the solar wind flows towards the tail of the magnetopause, it becomes supersonic and eventually super-magnetosonic (flow velocity larger than the phase velocity of the magnetosonic waves) due to the change of the physical conditions (Spreiter et al. 1966). In super-magnetosonic regime, corresponding to a Mach numbers of the order of (or larger than) unity, the vortices generated by the nonlinear development of the KH instability act as an obstacle in the flow and can induce the formation of shock structures extending well outside the region of velocity shear.

In this paper we review the main results reported in previous works (Palermo et al. 2011a,b) in which we have investigated the behavior of the KH instability in the transition from sub-magnetosonic to super-magnetosonic regimes in typical conditions observed at the Earth’s magnetopause flanks. We recall that the supersonic and/or super-magnetosonic regimes are in general defined with respect to the solar wind velocity, while here we are mostly interested in supersonic and/or super-magnetosonic Mach numbers calculated by considering the velocity difference between the KH vortices and the large scale flow (Miura 1992).

2 Numerical method

We adopt a two fluid description of the plasma dynamics. All the used equations, set in dimensionless form, are shown in Palermo et al. (2011a). Here we only note that, since we expect the formation of shocks, we adopt an adiabatic closure. Moreover, we consider a generalized Ohm’s law that includes electron inertia effects (Valentini et al. 2007). The mass ratio has been fixed as \( m_i/m_e = 64 \). In the code, all quantities are normalized to the ion gyrofrequency, the ion inertial scale length \( d_i = c/\omega_{pi} \) (with \( \omega_{pi} \), the ion plasma frequency and \( c \) the speed of light), and the Alfvén velocity \( c_A \). We consider a 2D, \( L_x \times L_y = 120 \times 60\pi \), box with 3D velocity and electromagnetic fields. We impose periodic boundary conditions along the \( y \) direction corresponding to the solar wind flow. We use open boundary conditions along the inhomogeneous \( x \)-direction. In this way, all the perturbations generated in the central region (where the dynamics develops) are free to leave the domain without reflection (Faganello et al. 2009). We achieve numerical stability using filters (Lele 1992). These filters also allow to resolve the properties of the shocks that develop in the simulations. The initial profiles are chosen to represent the magnetospheric and magnetosheath region in the left (\( x < 0 \)) and in the right (\( x > 0 \)) side respectively. Thus, the density and electron/ion (subscripts \( e/i \) respectively) temperature profiles are chosen as:

\[
n(x) = n_0 - \Delta n/2 \left[ 1 - \tanh(x/L_{eq}) \right], \quad T_{e,i}(x) = T_{0,e,i} + \Delta T_{e,i}/2 \left[ 1 - \tanh(x/L_{eq}) \right],
\]

where \( \Delta n = 0.9 \), \( \Delta T_e = 0.4 \) and \( \Delta T_i = 2.4 \) are the values of the density, electron and ion temperature jump between the magnetosheath (R) and the magnetosphere.
Fig. 1. Shaded isocontours of the density at $t = 143$ for $V_0 = 5$ in the $(x, y)$ plane.

(L) plasma, with $T_{0,e} + T_{0,i} = 1$ and $n_0 = 1$. Moreover, we take the transition layer width $L_{eq} = 3$. The magnetic field $B_0(x)$ is chosen such that the total pressure $P_T = (P_e(x) + P_i(x) + 0.5B_0(x)^2)$, is initially uniform in the $x$ (inhomogeneous) direction (with $P_e + P_i = (T_e(x) + T_i(x))n(x)$). We take the magnetic components $B_y(x) = B_0(x)\sin\theta$ and $B_z(x) = B_0(x)\cos\theta$, where $\theta$ is set to 0.02 to make the magnetic field almost perpendicular to the $(x, y)$ plane. We choose a reference frame (hereafter the simulation frame) moving at one half the solar wind speed and in which the flow velocity $V_{eq} = V_0/2\tanh(x/L_{eq})e_y$ takes the values $\pm V_0/2$ in the $y$ direction in the asymptotic magnetosheath and magnetospheric regions. In this simulation frame, we define a vortex Mach number as follows:

$$M_{vort}^{f,L/R} = U_{L/R}/c_{f,L/R}, \quad U_{L/R} = \|V_0/2 \mp V_{vort}\|,$$

(2.1)

where the different quantities are considered in the magnetospheric (L) or in the magnetosheath (R) side respectively, and where $U$ is the relative velocity of the vortex with respect to the flow, $c_f = (c_s^2 + c_A^2)^{1/2}$ is the fast magnetosonic velocity (with the sound velocity $c_s = [5/3(P_i + P_e)/n]^{1/2}$), while $V_{vort}$ is extrapolated from the simulations. This definition is similar to the “convective Mach number” used in Miura (1992). The equilibrium conditions give $c_{f,L} \approx 5.4$ and $c_{f,R} \approx 1.6$.

3 Numerical results

We present the results obtained by varying the flow intensity in the range $3 < V_0 < 7$. In our simulation frame, the KH vortices remain at rest for the case of a uniform density plasma. Otherwise, considering the case with a density gradient, the KH vortices propagate in the direction of the flow of the denser plasma. We recall that in our case the denser plasma is the one of the magnetosheath region (R). In case of incompressible plasma with a discontinuity in the density and velocity profiles ($L_{eq} \to 0$), we can estimate the vortex velocity as (Otto & Farfield 2000)

$$V_{vort}^{theor} = V_0/2(n_{0,R} - n_{0,L})/(n_{0,R} + n_{0,L}).$$

(3.1)

The most important result that we found in our simulations is shown in Table 1, where we compare the theoretical vortex velocity of Equation (3.1) and the one
Table 1. The theoretical vortex velocity of Equation (3.1) and the one extrapolated by our simulations, in the simulation frame, for different values of the solar wind velocity $V_0$.

<table>
<thead>
<tr>
<th>$V_0$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{vort}^{\text{theor}}$</td>
<td>1.2</td>
<td>1.6</td>
<td>2.0</td>
<td>2.4</td>
<td>2.8</td>
</tr>
<tr>
<td>$V_{vort}$</td>
<td>0.8</td>
<td>1.0</td>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
</tr>
</tbody>
</table>

extrapolated by our simulations, as function of $V_0$ in the simulation frame. Due to the compressible effects that become increasingly important for larger values of $V_0$ we found that the value of the vortex velocity $V_{vort}$ is lower than that predicted by Equation (3.1). Therefore, Mach numbers $M_{f,R}^{\text{vort}} \gtrsim 1$ are obtained for values of the solar wind velocity $V_0$ lower than that predicted by using $V_{vort}^{\text{theor}}$ instead of $V_{vort}$ in Equation (2.1). For instance, we observe that for the case $V_0 = 5$, the vortices propagate at $V_{vort} \approx 1.1$ corresponding to a magnetosonic vortex Mach number $M_{f,R}^{\text{vort}} \gtrsim 1$. At Mach number $M_{f,R}^{\text{vort}} \gtrsim 1$, vortices act as a super-magnetosonic obstacle in the plasma flow. Thus, in correspondence to each vortex, a shock structure develops outwards in the magnetosheath region. This aspect is shown in Figure 1 by means of the shaded isocontours of the plasma density at $t = 143$ for the case $V_0 = 5$. We observe the vortex structures in the centre of the box and the shock structures (strong density gradient: green to orange color) in the magnetosheath side. Since $M_{f,L}^{\text{vort}} \ll 1$ no shock is observed in the magnetospheric side. In this regime the physical conditions, found inside the vortices (for instance uniform density), make them stable with respect to secondary instabilities. We note however that secondary instabilities can develop downstream from the shocks near the vortex boundaries (see Palermo et al. 2011b). In conclusion, we conjecture that the shock structures associated to the nonlinear KH vortices, that we have put in evidence in our simulations, should be observed in satellites data.

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References

Valentini, F., et al., 2007, J. Comp. Phys., 225, 753